

Computers and Calculators in Mathematics Education

An Honors Thesis (Honors 499)

by

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A handwritten signature in cursive script, reading "John W. Emert", is positioned above a solid horizontal line.

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Abstract

Computers and calculators have made an impact upon all aspects of modern society, including school systems. Some schools have accepted this new technology without hesitation, but others have not. There are a wide range of opinions about the way computers and calculators have been integrated into schools. This thesis will examine a variety of software packages, the ways that experts feel about integration, and also look at current teachers' perspectives.

For decades, even centuries, mathematics has been taught with the use of only a pencil and paper. Students became proficient at mathematics, and some even went on to create new and better methods of problem solving without the use of modern technology. Today the most common forms of modern technology in mathematics education are the computer and the calculator. Computers were originally designed to solve complex mathematical problems which would have taken days or even months to solve by traditional methods. [20] Modern calculators evolved out of early computers and are currently accessible to almost everyone. It would then appear that mathematics and computers go hand in hand; however, some teachers feel that computers should not be integrated into the mathematics classroom or even into the schools at all. These people are not necessarily against the use of computers, but they are apprehensive about computer integration because of previous failed attempts. [18]

One of the greatest fears associated with computer integration is that the basic skills will be overlooked. [18] These people feel that the calculator and the computer will replace the real understanding of mathematics, but what *is* the real understanding of mathematics? Is it the logic and higher reasoning skills gained through mathematics, or is it simply the addition, subtraction, multiplication, and division of large numbers. Years ago it was necessary to do difficult problems with a slide-rule, log tables, or simply by pencil and paper, but with the technology of today there is no need for this type of work. It is possible that the basic skills of mathematics have changed over the years. The "basic skills are important, but they must be the skills students will need for life and work . . . in this era." [7] Ideally, mathematics teaches logical thinking skills and applications for use in everyday life. It is difficult for some teachers to simulate this ideal into their classrooms, and this is exactly where computer integration can help.

Few will deny that the computer can offer students many opportunities that would otherwise be overlooked. Nevertheless, some criticism is voiced about the way computers have been placed in the schools. Most teachers are "in favor of computer education, although it's not clear that the present trend is appropriate." [18] Most experts would agree that previous attempts at computer integration fell far short of desired outcome. In past integration attempts the main focus was on the computers themselves. School systems' first priority was to get the best, most up-to-date software and hardware. It was assumed that teachers would be able to use this equipment just as easily as they use an

overhead or filmstrip projector. Untrained teachers attempted to use this new equipment, but most soon became discouraged and gave up. Previous failure makes many leery of another attempt at computer integration; however, businesses, administrators, and educators have learned a great deal from their mistakes. [10]

One of the most important lessons learned was that untrained teachers cannot be expected to immediately integrate calculators and computers into their classrooms. Instead of the focus being on the machines, attention should be placed upon the people involved in the learning process, the teachers and the students. Most agree that computers can make a useful contribution to the current curriculum, but "if computers and calculators are to be important to school-based education then it is also important that they are used wisely." [18] They must be handled by people who understand what teaching is all about, both with and without computers. Teachers need to know when computer integration is most useful and not just use it as a pastime for students. The importance does not lie in the machines themselves, but rather in the training of the personnel involved. As Snyder states, "the teacher is by far the most important element in an effective learning situation." [18]

Despite the disappointing history of computer integration in schools, computers are a part of the world in which we live. School systems are designed to prepare young people for their futures: college, jobs, etc. Since computers have infiltrated practically all areas of daily living, it is critical that computer integration in the schools continue. According to Thomas A. Romberg, chairperson of the NCTM Commission for School Mathematics, ". . . it is imperative that schools play an active part in preparing students to live in a world in which more and more functions are being performed by computers." [2] Ideally, computer integration should occur in all areas of the school curriculum from science to music to history, though some subjects lend themselves more easily to integration than others. The mathematics classroom is a logical location in which to start the integration process.

As Kelman notes, there are many reasons why the mathematics classroom is one of the best choices for computer integration. [7] First, as previously mentioned, the computer was originally designed to do complex mathematical computations. Since the computer has its roots in mathematics, its growth could continue through the mathematics classroom. Second, during the 1960's a group of mathematics teachers were among the first to participate in

the teaching of computer programming the mathematics classroom. The brightest mathematics students were given added instruction in computer programming in addition to the traditional curriculum. Mathematics teachers were also among the first to use CAI (computer aided instruction) in their classrooms. This was one of the first steps toward integration rather than separating computers into a different class. Finally, the first educational games were in mathematics. Pong, designed for entertainment, could be used to help students develop a basic understanding of angles. Games which aided young children in learning multiplication tables or addition of fractions were among the first types of educational games available on the market. [6]

In an effort to determine the attitudes of current teachers toward the use of computers and calculators in the mathematics classroom, I recently conducted personal interviews with four teachers. Teacher A is from a small rural school which houses about 600 students from seventh to twelfth grade. She is in her early thirties and teaches most of the middle school age students. Teacher B is a bit older, mid 40's, and teaches high school age students. She is from a large city school where the enrollment is approximately 2,000 from seventh to twelfth grade. In contrast, Teacher C comes from another rural school which is noted to be economically superior to the school of Teacher A. Teacher C is in his mid 50's and in addition to teaching at the high school level, he is also employed by Ball State University to teach an intermediate algebra course. Finally, Teacher D is from a school very similar to that of Teacher A. He is in his late 50's and teaches the upper level mathematics classes including pre-calculus. All four of these teachers were asked the same basic questions and a complete summary of the interviews can be found in Appendix 1.

The idea of using games as a learning tool is still very prevalent even at the secondary school level. However, there is a great need for more advanced software. [6] Children are intrigued about new things and anxious to try them out. This innate interest can be expanded upon by the use of computer integration. Despite the widespread use of computers in society, computers are not a common occurrence in the household; therefore, most children are excited at the opportunity to use computers in school. As Teacher A noted, computers and calculators are different, and variety is what makes a child want to learn.

Computers have evolved a great deal since Eliac 1, the first computer, whose physical components filled an entire room. [20] In the same respect, computer integration curricula have also changed remarkably over the past ten

years. The first computer curricula were designed to look and feel like workbooks and textbooks. [7] This original trend is where drill-and-practice exercises began. Drill-and-practice exercises are still in great use today. These types of computer exercises offer a number of advantages over the traditional workbook methods. First, drill-and-practice exercises that would take the classroom teacher hours to prepare and grade can be done in a minimal amount of time with the use of computers. The computer also allows for a student to receive immediate feedback for his work. This type of feedback "can nip a problem in the bud, keep the student from wasting lots of time and energy, and prevent bad habits from becoming ingrained." [2] Some students work at a faster and/or higher level than others. Computer drill-and-practice sets allow a student to work at his or her own pace, free from peer ridicule or embarrassment. These exercises also allow a teacher more time to teach instead of being bogged down by the more time consuming aspects of the job.

Drill-and-practice exercises are not the most exciting nor the best use of computers in the mathematics classroom. Many teachers today do not consider these exercises to be proper integration. As all four of the teachers who were interviewed mentioned, computers and calculators should be used more as exploration tools instead of providing simplified busy work. Teacher C felt that as soon as students have shown that they are competent in the basic two- or three-digit whole number multiplication and division, calculators and computers should be allowed--even as early as the elementary grades. Teacher A even felt that students may become disinterested in computer applications if drill-and-practice exercises are all they are exposed to.

More practical forms of computer integration occur in the use of computers and calculators as learning tools. [7] Computers can eliminate many of the tedious aspects of arithmetic. The ultimate goal of mathematics education is to assist students to develop logical thinking skills rather than just reiterate a formula. [11] Many students have negative reactions toward mathematics because they simply can't get the "right" answer. They do not realize that many times the procedure can be correct, and they only have a small arithmetic error. Computers can eliminate some of the negative attitudes that develop by this process. Also, most real world problems do not contain simple whole numbers. In traditional workbook exercises the problems were usually adjusted so that the answers worked out easily by sacrificing the realism of the problem. The computer and calculator allow for more realistic problems to be developed yet still

remain easily solvable. [11] By relieving students from the tedious aspects of arithmetic, computer integration can offer opportunities for a more concept based classroom.

Computers and calculators can also be used as exploration tools. As Teacher A described, exploration involves allowing students to learn at their own pace and to try new things without outside pressures. For example, exploration is a very important aspect of graphing. [19] Once students learn how to graph basic linear equations, they can use computers and calculators to understand higher degree equations without the complex mathematics usually associated with these graphs.

Graphing calculators are becoming more common in secondary mathematics classrooms. These calculators offer a less expensive option to entire computer systems. Their graphing capabilities are very similar to that of some software packages. The graphing calculators "require student understanding and skill in the use of algebraic logic for the entry of expressions, function rules, equations and inequalities." [9] The use of graphing calculators would in no way take away from the understanding of basic algebraic functions. These calculators actually enhance students' learning by making the subject more concrete. In a study done by Ray Hembree and Donald Dessart, research confirmed the theory that the use of calculators can even elevate a students' attitudes toward mathematics and their self-concepts. [9] However, one of the disadvantages is that these calculators are limited in scope compared to a full computer system. Two of the more common software packages which include extensive graphing capabilities are the Geometric Supposer and Logo, and these are described below.

The Geometric Supposers are "computer based slaves that will do your constructions for you." [2] These programs allow students to make conjectures and find out if they are true statements through computer-based constructions. By freeing students from formal construction the student can continue on to a high level of understanding without the complex mathematics. The Supposers have functions which range from angle bisecting to constructing tangent lines of circles. The Geometric Supposers offer a solution to this problem. The program does each construction movement separately so that students can learn from observing the computer. As Yerushalmy observes, "geometry teachers have always faced the dilemma of having to instill in their students an

appreciation of deductive mathematical systems while at the same time offering them an opportunity to create mathematics." [21]

Logo is a programming language, but it is not as complex as most. Logo can be thought of as telling a turtle which direction to turn and how far to crawl. At the high school level Logo can be used to enhance students' logical thinking and spatial skills. [1] Even at the collegiate level this program can be used in complex geometric constructions. Logo is a very systematic language that is easy to understand, yet it can be used to develop higher level geometric concepts. Its capabilities range from programs which draw a simple square to ones which include numerous interlocking procedures. Logo seems to be especially useful in the elementary curriculum. Many teachers feel that a child's first experience with geometry should not be so formal, but should actually be a more informal study of physical shapes and their properties as a way to develop students' intuition and knowledge about their spatial environment. [11] Logo offers the opportunity for children to explore geometry without formal proofs.

The Geometric Supposers and Logo are both excellent programs for the high school mathematics classroom. Each has a wide range of capabilities to assist students at all levels. Both have deep roots in geometry, in particular logic. Logo does appear to be more versatile since it can be used at any grade level from elementary through college; however, the Supposers are better for people with less computer experience.

Logo also emphasizes another use of computers. Programming can be a beneficial experience to the mathematics student. As Teacher B noted, programming teaches a student logical thinking skills, the foundation of problem solving. Most of the current programming software can be used in either secondary or collegiate mathematics. This software ranges from Logo, which consists of relatively simple commands to Mathematica, a much more sophisticated package. Most mathematical programming software lies in between these two extremes.

Mathematica is one of the most complex mathematical software packages on the market today. What it lacks in simplicity it makes up in versatility and power. Mathematica is used in many colleges across the nation, and some high schools have started to use it in their higher level classes. It allows students to input very difficult integration or factoring problems - as well as many other types- and see an answer in a relatively short time. This is only part of what Mathematica can do. [8] It is best known for its superb graphing capabilities.

Without knowing how to graph many higher order expressions, students can learn patterns which are common to a particular family of graphs. Mathematica is a good program to use as an exploration tool; however, it does seem to work best when the students have had some computer experience and have a firm grasp of beginning concepts of calculus.

Math Cad and Derive are software packages similar to Mathematica, except that they are not as powerful. Both could be integrated very easily into the secondary mathematics curriculum. Students could use such software to explore a variety of mathematical concepts, and also as tools to check their work. Both packages have a wide variety of mathematical functions, and both are more affordable than Mathematica. These two software packages are accompanied by well-written manuals which someone with little computer experience can understand.

Since everyone is affected by computer technology it is easy to see that students who have the benefits of computers will be far ahead of those who do not. There are many other software packages on the market which range from mathematical games to complex programming. For any mathematical setting, there is a software package which could enhance good teaching techniques, but teachers and administrators must be willing to work together to educate teachers on various integration possibilities. There is not one easy way to integrate computers and calculators. Teachers must be allowed to try and possibly fail in order to discover what technique works best for them. Computers have already encompassed the collegiate world of mathematics as well as that of industrialized world. It is time that schools meet the demands of the world by utilizing computers and calculators in the mathematics classroom.

It is evident that students' curricular needs have changed over the past twenty years. Textbooks, worksheets, and chalkboards are no longer sufficient tools to prepare students to function in today's "high-tech" society. Computer integration needs to occur across the curriculum at both the elementary and secondary levels. Mathematics seems to be one of the best starting points for that integration. Although only a few options were mentioned here, the possibilities that computers and calculators offer are as numerous as there are teachers. Teachers who have proper training, funds, and imagination are the key to successful computer integration. The students of today are the leaders of tomorrow, and it is only fair that schools teach them what they truly need to know.

APPENDIX #1

Current Teacher Interviews

This appendix contains summaries of interviews with current teachers that focused on the issue of computer/calculator use in the mathematics classroom. All of the teachers interviewed were aware of the NCTM Curriculum Standards. Since this was a year for mathematics textbook adoption, all of the teachers were using the Standards as a guideline in choosing textbooks.

Teacher A

Teacher A is from a small rural school which houses about 600 students from seventh to twelfth grade. She is in her middle thirties and teaches most of the middle school age students.

Teacher A is greatly in favor of technology being integrated into all areas of the school. She does not have much computer experience, but is very willing to learn. She would like to use computers for demonstration purposes as well as have more advanced graphing calculators or computers available for student use; however, her school system does not have these resources. She feels that students are more anxious to learn in an environment where there is variety. Computers/calculators could add to the this variety in addition to the options that a teacher has available.

She is not in favor of using computers strictly for drill-and-practice exercises, but rather thinking that only small amounts of this type of exercise should be used. She feels that students will become disinterested in computer applications if drill-and-practice exercises are all they are exposed to. She believes that students should not be bogged down by "busy work." She also does not advocate the use of computers as a bonus incentive. She feels that this would only benefit the faster students who could finish their work quickly. If given the opportunity, she would like to use computers as an exploration tool. In this manner she could have students investigate and learn about such things as higher level graphs without necessarily knowing the specifics of plotting these graphs by pencil and paper. She said, "after all, in the real world computers do all of this type of work for you." Ideally, she would have at least one computer for every two students in her classroom.

The school at which she teaches does require a computer course for graduation, but the teacher who currently teaches this class is not formally trained in computers. Presently, a business teacher who has taught herself about the computer is teaching the class. Teacher A indicated that she, along with most of the faculty, would be very interested in learning more about computers and integration, but due to lack of funding and time most cannot afford to do so.

Overall, Teacher A believes that a computer can be as useful to the mathematics curriculum as pencil and paper, but she is able to teach without them. Nevertheless, she believes that this lack of computers/calculators will only hurt the students in the long run.

Teacher B

Teacher B is in her middle forties and teaches high school students. She is from a large inner city school where the enrollment is approximately 2,000 students from seventh to twelfth grade.

Teacher B is principally involved in the computer education of her school system. Despite the fact that she has no formal training in computers, she appears to be the "resident expert." She has attended many seminars and workshops to further her knowledge. She teaches many of the programming courses which are offered at her school.

She advocates the use of calculators in higher level mathematics classes such as calculus or trigonometry. She suggests very limited calculator use in classes such as general math or first year algebra, because "these students need to understand the basic principles of mathematics before they use computers/calculators on a regular basis." The math department owns approximately 60 graphing calculators, but these are used mostly by the calculus classes.

Teacher B feels that programming is a very worthwhile activity for students. She stated, "students gain logic and math skills through programming." However, there is not a great deal of student interest in this type of class, so the class is often canceled. Little integration occurs because the teachers have not been properly trained on the potential that computers have. Stand-alone units are available for demonstration purposes but remain relatively unused. Teacher

B does not recommend drill-and-practice exercises. Overall, she feels that "a computer/calculator should only be used in situations where it would be a more efficient means of solving a problem than paper and pencil would be."

Teacher C

Teacher C is from a rural school which is noted to be economically superior to the school of Teacher A. Teacher C is in his middle fifties and, in addition to teaching at the high school level, he is also employed by Ball State University to teach an intermediate algebra course.

Teacher C did not feel that he was very knowledgeable about computers in education, yet he realizes their importance in today's society. "If computers are in the society, and schools are to be preparing students to be productive members of society, then it is only logical that the school should be teaching children about computers and their uses." Since he feels undereducated about computer/calculator uses, he does not teach with computers. However, he feels as though more teachers should use computers in their classrooms.

He did feel that calculators should be used as soon as the basic skills are mastered, possibly as early as the elementary grades. He advocates calculator use in both lower and upper level mathematics classes. He feels that calculators would allow students to move beyond computations. He did not feel that calculator use should be restricted in any way by stating, "calculators may only serve as moral support but at least that will give students the needed confidence to succeed in not only mathematics but other classes as well."

Teacher C did not feel adequately informed to discuss computers and their pros and cons. He said that integration would be very beneficial for students, but that he does "not feel qualified to get the ball rolling." He does realize that computers are a necessity for most careers and colleges.

Teacher D

Teacher D is from a school very similar to that of Teacher A. He is in his late fifties and teaches upper level mathematics classes including pre-calculus.

Teacher D believes that calculators should be used only after students have mastered the basic skills of mathematics. He stated that calculators with graphing capabilities should only be used in the upper level mathematics classes because he feels that students in the lower level classes would not have

the capacity to understand the graphs or the graphing process; however, a basic four function calculator should not be restricted once students have demonstrated their competence in these areas.

Teacher D understands that there is a value in computers, yet he does not use them in his classes. He feels that "too much time is involved in getting to the computer lab and setting up the computers." He implied that if the computers were more accessible, more of the teachers would be willing to try computers as a means of instruction or for student use. He believes that students would benefit more if computers were used for independent studies. He did not feel that computers would do any good without the proper software and adequate student motivation. He agrees that students can learn without computers, yet he feels that they are missing out on a needed skill in today's society - computer literacy. He believes that computers are misused as replacements for paper and pencil drill-and-practice exercises and as bonus incentives to students. In closing he stated, "students would benefit the most from complete computer integration."

Appendix #2 Selected Software Reviews

Mathematica

Mathematica is an excellent program for higher level mathematics. It enables students to graph three dimensional shapes that would otherwise be very difficult or impossible to imagine. It also allows students to work very complex mathematical equations with ease. This permits students to focus on the concepts involved rather than worrying about the arithmetic involved. An instructor would be able to introduce a number of nontraditional topics such as fractals and various surface areas. However, the students seem to benefit more if they already have some understanding of the topic before using Mathematica. It does not appear to be a good program for introducing topics. Mathematica is very powerful, but it is a very case sensitive program. Overall, Mathematica is not user friendly. One must be able to decipher codes in order to figure out mistakes. Mathematica includes excellent documentation that is easy to understand for a person with minimal computer background. The program can perform calculations with a high degree of accuracy. I do feel that a person would need some previous computer experience and a fairly good grasp of the mathematical topic at hand in order to utilize Mathematica's highest potential.

Derive

Derive is a type of programming software. Like Mathematica, it does not act as a tutorial. On the contrary, Derive functions like a high powered calculator. It has many options from factoring and simplifying to differentiation and integration. Derive simply requires one to input an algebraic equation or expression and then choose the desired command. Derive is very versatile, it can even plot three dimensional shapes. Its algebraic and calculus accuracy are excellent while some of the graphs are rather crude.

Derive would be a good program for student exploration at the high school or college level. In my opinion, it may even be suitable for some middle school students who are just learning algebra. It would allow students to work with some higher level mathematics without difficulty. The accompanying guide is very clear and easy to read. I feel that someone with little computer experience could use this software; however, one would need to have a good grasp of basic mathematical concepts in order to receive the full benefits of this software.

Math Cad

Math Cad is similar to Mathematica in that it is not a tutorial. It is a program which can help students better understand unclear topics. One must be able to input directions and formulas for the program's use. It contains excellent help menus, and it is very user friendly. Math Cad is not case sensitive in that upper and lower case letters are not differentiated.

This program would be excellent for writing a mathematical paper. It allows for most of the traditional mathematical symbols that cannot be found on a regular word processor. Math Cad is very precise in its information. The computations performed can be carried out to a specific number of decimal places, and the mathematical symbols such as fractions actually look like fractions.

I feel that Math Cad has more limitations than software such as Mathematica or Derive. I do not feel that it is flexible enough for use at the secondary level.

Geometric Supposer

The Supposer series consists of four programs: The Geometric Presupposer: Points and Lines, The Geometric Supposer: Triangles, The Geometric Supposer: Quadrilaterals, and the Geometric Supposer: Circles. All of these have generally the same characteristics. The Supposer is an exploration series where the student can make a conjecture and then test it out in a variety of situations to see if he was correct. The program will do any construction one desires much faster than could be done by hand. In addition, the Supposer is much safer than a compass for younger students. The Supposer also gives students who do not know how to do actual constructions an opportunity to learn by observing the computer as it does each construction step by step just as one would with a compass and straightedge. This program enables students to discover theorems instead of just reading and accepting them. It helps students by offering a hands-on type of experience in geometry that may otherwise be unavailable.

The program is fairly user friendly. It does not allow students to do constructions that are not possible, but it does not give a message why the construction cannot be completed. It only ignores the command which can become a very frustrating experience for students. The figures are somewhat crude, but a color monitor allows the figures to be more understandable. There are help commands at every level, and the labeling is very clear. The output

that students receive is very accurate. It is also easy for a student to erase or fix a mistake. A teacher may use the materials provided, design his own supplement, or simply allow students to explore on their own.

I feel that this is an excellent program for all students. It does not require any previous computer programming experience. This program gives clear command that a novice could understand. In addition, only the basic ideas of geometry, such as shapes and definitions, need to be understood before using this program. It is very adaptable, and allows for students to explore and create at their own pace.

Logo

Logo is a unique program in that it can be used from the elementary up through the college level. At the elementary level, Logo is a superb introduction to geometry. It allows for young children to explore and be creative. Simply by drawing a picture a child would have to recognize the various shapes and also be able to tell the turtle how to move to accomplish these shapes. At the secondary and college levels, Logo can also be used in the same manner for slower students, but for those who have a fairly good knowledge of geometry, it can be used to teach logic skills and dissection of complex figures.

Logo is very user friendly. If a mistake is made in the programming it will direct the user to the exact spot of the mistake; however, if a logic error occurred and the figure does not look as desired then it is up to the user to find the mistake. This program has many options such as size, color, and even sound which makes it more fun than other programs. The graphics are clear and accurate. Unfortunately, no help screen is available, but the literature which accompanies Logo is simple to understand.

Logo is like Mathematica and Math Cad in that it is a programming based application; however, it is much simpler to use. Once a child learns the basic commands, he can easily program in Logo. It is as simple as telling a person to take twenty steps forward and then turn right. I feel that Logo is one of the most versatile and useful programs on the market. A teacher could integrate Logo into a variety of lessons with little difficulty.

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